A comparison of the unit step responses is shown in Table 1. If one wishes to design a PID controller for a system with flexible appendages (such as Space Telescope solar panels), appendage dynamics would be characterized in modal coordinates, such as normal modes. Their inclusion increases the order of the characteristic equation, but the referenced parameter plane technique still could be applied to establish values for modified gains *a, b, c*. The resulting analytical complexity is being checked now by the author to determine if this complexity masks the usefulness of the technique described. If so, the technique still is useful for initial selection of gain values and sample period for preliminary design purposes.

Conclusions

A technique to aid the control system engineer in his selection of numerical values for satellite onboard digital computer gains and sample period has been presented. It is postulated that the technique may be extended to handle larger-order system to increase the fidelity of the system that is represented analytically.

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Yaw Induction by Mass Asymmetry

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Nomenclature

I_t = transverse moment of inertia of the shell
I_x = axial moment of inertia of the shell
K_j = absolute value of the jth modal arm, rad ($j=1, 2, 3$)
l_ϵ = distance of unbalance along the axis of symmetry in front of the center of mass of the shell
m_ϵ = mass of unbalance
r_ϵ = distance of unbalance off the axis of symmetry
t = time
$\tilde{\alpha}$ = angle of attack in a nonrolling missile-fixed system
$\tilde{\beta}$ = angle of sideslip in a nonrolling missile-fixed system
λ_j = damping rate of the jth modal arm ($j=1, 2$)
ϕ = roll angle of the shell (taken as zero at time zero)
ϕ_j = orientation angle of the jth modal arm ($j=1, 2, 3$)
$\dot{(\ )}$ = derivative of ( ) with respect to time
$(\ )_0$ = value of ( ) at time zero

Introduction

THE need for controlling launch yaws during development flight testing has been generally recognized by most aeroballisticians. A particularly dramatic example of this need is given by the 155-mm M483 shell. In January 1974, 20 of the M483's were fired at transonic launch Mach numbers and seven flew to less than 65% of full range.[1] During the remainder of 1974, a substantial engineering effort led to a modified version of the M483 which apparently did not have the transonic instability observed in January. Final proof tests of this improved version were planned for the following winter. During these tests, unmodified M483's were fired as controls, with the unexpected result that 48 successive unmodified M483's achieved full range, and the first short occurred for the 49th round![2] Since the shorts occur only when the launch maximum angle of attack exceeds 5°, the tube-projectile combination of 1975 apparently gave much smaller launch angles than the combination used in 1974. Since natural gun launch was, therefore, not a very reliable test, the validity of the M483 modification had to be established by artificially inducing yaws up to 16° and observing the resulting rapid damping of the angular motion.

Yaw usually is induced by means of modified muzzle brakes. Although this technique is reliable for moderate muzzle velocities, it can damage the shell or gun at high muzzle velocities. In this Note, we shall show how the introduction of a mass asymmetry can be a convenient yaw-induction technique.

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Theory

As is shown in Refs. 3 and 4, the effect of a small dynamic unbalance is to add a third mode of oscillation to the usual two modes present in the shell's angular motion. This third mode causes a circular motion with frequency equal to the shell's spin and a magnitude equal to the angle between the unbalanced shell's normal axis of inertia and the balanced shell's normal axis of inertia. For the unbalance introduced by a small mass m_ϵ located a distance r_ϵ off the axis of symmetry and a distance l_ϵ along the axis of symmetry in front of the shell's center of mass, this angle is

$$K_3 = m_\epsilon l_\epsilon r_\epsilon / (I_t - I_x) \tag{1}$$

The complete tricyclic equation for pitching and yawing motion is

$$\tilde{\beta} + i\tilde{\alpha} = K_1 \exp(i\phi_1) + K_2 \exp(i\phi_2) + K_3 \exp[(i(\phi + \phi_{30})] \tag{2}$$

where

$$K_j = K_{j0} \exp(\lambda_j t) \qquad (j=1,2)$$

$$\phi_j = \dot{\phi}_j t + \phi_{j0} \qquad (\dot{\phi}_1 > \dot{\phi}_2, j=1,2)$$

With the reasonable assumption of small initial angle and the approximation of small angular velocity, Ref. 4 shows that

$$K_1 = [(\dot{\phi} - \dot{\phi}_2)/(\dot{\phi}_1 - \dot{\phi}_2)] K_3 \tag{3}$$

$$K_2 = [(\dot{\phi} - \dot{\phi}_1)/(\dot{\phi}_1 - \dot{\phi}_2)] K_3 \tag{4}$$

For gyroscopically stable shell, the frequency factors in Eqs. (3) and (4) usually exceed eight, and thus an unbalance angle of 0.5° can cause at least 4° in the fast and slow motion

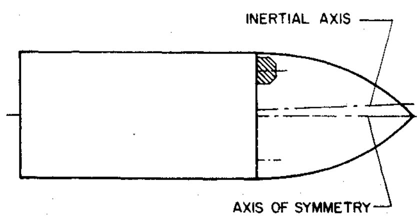

Fig. 1 Schematic of the shell with asymmetric mass.

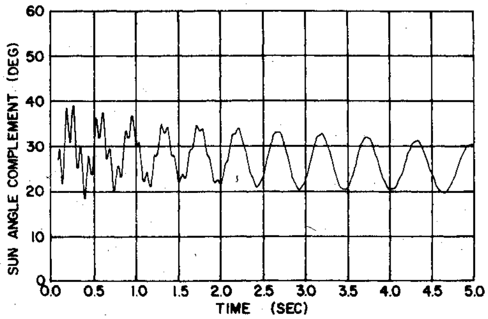

Fig. 2 Sun angle complement vs time for mass-asymmetric shell.

modes and an initial maximum angle of attack of at least 8°. This property of dynamic unbalance is very valuable for its use as a yaw-induction technique. In a recent paper,[5] Hodapp showed that a random small dynamic unbalance induced by manufacturing inaccuracies could be a significant source of dispersion. In this paper, we are proposing the use of intentionally large dynamic unbalance as part of a critical test procedure.

Experimental Results

The first use of this technique was for flight tests of an 8-in. shell. The mass asymmetry was introduced by adding a 1.75-kg mass just forward of a bulkhead between the cylindrical body and the ogival nose (Fig. 1). More precisely, this mass was located 32.5 cm from the center of mass of the shell and 6.0 cm off its axis of symmetry. The predicted cant of the normal axis of inertia was $K_3 = 0.0112$ rad $= 0.64°$.

The measured $\dot{\phi}$, ϕ_1, and ϕ_2 were 106, 13.3, and 2.7 Hz, respectively. These values in Eqs. (3) and (4) yield an estimated initial maximum angle of attack of 11.8°. The actual angular motion of the shell with this asymmetric mass was measured by yawsondes[6] and is given in Fig. 2. The first maximum angle of attack was 9.3° for this flight. Thus we see that this technique can be used to insure a reasonably large initial yawing motion.

An early version of this shell showed a rapid growth of the amplitude of its fast mode of oscillation. Firings of an improved version showed essentially *no* fast mode of oscillation. As a result of our M483 experience, it was necessary to make some stimulated launches to insure that the amplitude of the fast mode of oscillation will damp to zero from initial nonzero values. The dynamic-unbalance-stimulated launch induced an initial fast-mode amplitude of 4.6°. This fast-mode motion, however, damps completely within 4 sec, and the remaining slow-frequency motion then slowly decreases to the slow-mode limit cycle. This behavior, which is shown clearly in the enlarged scale of Fig. 2, gives us strong evidence for *no* fast-mode instability associated with this improved projectile.

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